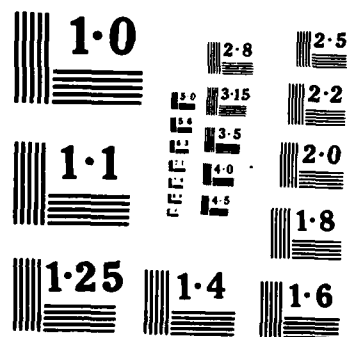| 8a. NAME OF FUNDING/SPONSORING ORGANIZATION | 8b. OFFICE SYMBOL (If applicable) | 9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER | | | |
|---|---|---|---|---|---|
| AFOSR | NM | Grant No. AFOSR 84-0269 | | | |
| 8c. ADDRESS (City, State and ZIP Code) | | 10. SOURCE OF FUNDING NOS. | | | |
| Bldg. 410 Bolling AFB,DC | | PROGRAM ELEMENT NO. | PROJECT NO. | TASK NO. | WORK UNIT NO. |
| 11. TITLE (Include Security Classification) (Please see reverse of page) | | 6.1102F | 2304 | A5 | |

| 12. PERSONAL AUTHOR(S) Shen-Neng Chiou and Victor O.K. Li | | | | |
|---|---|---|---|---|
| 13a. TYPE OF REPORT Reprint | 13b. TIME COVERED FROM _____ TO _____ | 14. DATE OF REPORT (Yr., Mo., Day) | 15. PAGE COUNT | |
| 16. SUPPLEMENTARY NOTATION IEEE INFOCOM, Miami- Florida, April 7-10, 1986 | | | | |

| 17. | COSATI CODES | | 18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number) |
|---|---|---|---|
| FIELD | GROUP | SUB. GR. | Network reliability, multistate models, multimode components, message delay. |
| | | | |
| | | | |

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

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| 20. DISTRIBUTION/AVAILABILITY OF ABSTRACT | 21. ABSTRACT SECURITY CLASSIFICATION |
|---|---|
| UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS ☐ | UNCLASSIFIED |
| 22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Victor O.K. Li | 22b. TELEPHONE NUMBER (Include Area Code) (213) 743-5543 | 22c. OFFICE SYMBOL AFOSR/NM |

UNCLASSIFIED   (Item no.11)

## RELIABILITY ANALYSIS OF A COMMUNICATION NETWORK WITH MULTIMODE COMPONENTS

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| DTIC TAB | ☐ |
| Unannounced | ☐ |
| Justification | |

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# Reliability Analysis of a Communication Network
## with Multimode Components[1]

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### Abstract

This paper presents a model to calculate the reliability of communication networks with multimode components. Previous research on network reliability has focused on models in which each component may be in one of two modes, namely, operative or failed. In reality, a component may undergo degradations in performance before a complete outage, and will therefore operate in more than two modes. Traditional network reliability measures, such as the probability that a pair of nodes is connected, are not meaningful in a multimode model. Therefore, the mean message delay of the network is defined as the performance measure. An exact calculation of this reliability measure is not feasible due to the large number of network states, corresponding to network components being in different modes. We have developed an approximation method to calculate this reliability measure. This method requires us to work with the states of the network in order of decreasing probability. An algorithm ORDER-M is developed to generate these states in the proper order.

## 1. Introduction

In analyzing the performance of a communication network with unreliable components, it is usually assumed that each failure-prone component can be in one of two modes, either operative or failed. In the operative mode, the component can handle load at its capacity, while in the failed mode, it is not available at all. Network reliability measures such as the probability that a given pair of nodes are connected, the probability that all pairs of nodes are connected, the expected number of communicating node pairs, and so on may then be defined. Numerous algorithms have been proposed (see, e.g., [1]) for computing the aforementioned network reliability measures. However, the computation time is enormous except for small-size networks. In fact, it has

been proved that the exact computations of these reliability measures are NP-hard [2] [10]. Other researchers [3] have proposed an approach that enumerates all possible network states and calculates the quantities of interest in each state. The reliability measure of interest is then calculated as a weighted ( by the probability of that state ) average of these quantities. Again, this approach is impractical in general because the number of network states grows exponentially, i.e., if there are n unreliable components, there are $2^n$ states for the network. We therefore seek an approximation method to minimize the computation time. In [9] an approximation approach is proposed. To find a reliability measure, only the most probable states of the network are analyzed. The idea is that when the states considered account for a large fraction of the state space (in terms of probability), we can get a good approximation to this reliability measure. This approach is practical because some of the network states may have very small probabilities, and the network rarely operates at these states.

While two-mode component models have received a great deal of attention, in reality, a component may operate in one of N ($N \geq 3$) modes, i.e., a component may undergo degradations before a complete outage. For example, if a radio channel is being jammed, one may try to combat this jamming by using a more powerful error-correcting code, with a corresponding reduction in the effective data rate or channel capacity [11]. Therefore, the channel will operate in multiple modes, with each mode corresponding to a different channel capacity. In [5] El-Neweihi, et. al. consider an N-state system which is a function of N-state (-mode) components and they are interested in the structure of the system function. They have not, however, indicated how one may define the N states of the network in a meaningful way in terms of the N modes of each link. In this paper, a multimode model for network reliability will be presented and used to compute some properly chosen reliability measures.

## 2. A Multimode Model

We assume a point-to-point network in which only links may fail or degrade and nodes are failsafe. The modes of a link correspond to different communication capacities of that link, i.e., each mode corresponds to a different capacity. Therefore, when a link is not operating

at its full capacity, it may operate at a lower, degraded service rate. Each mode of the component is associated with a known probability. There are M links in the network and N modes for each link, called modes $0, 1, 2, \ldots, N-1$. (We can handle components with less than N modes by defining the probabilities of extraneous modes as zero.) The jth mode of link i is denoted $C_i^j$, and the associated probability is denoted $p_i^j = Pr(C_i^j)$. We further assume that all link failures are independent. Note that there is a constraint for each i:

$$\sum_{j=0}^{N-1} p_i^j = 1, \quad i = 1, 2, \ldots, M$$

Given the network topology and the set of probabilities for link modes, we want to define a quantity to measure the reliability of the network. If we want the probability that a given pair of nodes are connected by a path of certain capacity, we may transform this problem to a typical problem in the two-mode model by calling the link modes which have at least that capacity as operative and other modes as failed. Therefore computing the probability of the existence of such a path may be obtained by the algorithms described in [1]. We therefore seek other performance measures which are meaningful for multimode models, and which require solutions significantly different from those for two-mode models. There have been attempts to use the probability distribution function of the maximum flow between a given source-destination pair as the performance measure in the multimode model. For example, Evans [6] studied this measure for a network in which the capacity of any edge is a finite-state integer-valued random variable. Kulkarni and Adlakha [8] considered a similar problem on (s,t)-planar networks with exponentially distributed link capacities. In this paper, the network mean message delay is chosen as the reliability measure since it is related to the degradation in channel capacity for the links in the network. With a given node-to-node traffic requirement, we are interested in the probability that the network can support the traffic with a given average delay. When the mean message delay is greater than this threshold at a given network state, we say that the network cannot satisfy the traffic requirements at that state. Hence we need to compute the probability that the mean network message delay does not exceed some finite value. For each state of the network (characterized by a set of link modes), we can compute the mean message delay T using Kleinrock's model [7]:

$$T = \frac{1}{\gamma} \sum_{i=1}^{M} \frac{\lambda_i}{\mu C_i - \lambda_i}$$

where $\gamma$ is the sum of the arrival rates between all pairs of nodes. $1/\mu$ is the mean packet length, $\lambda_i$ is the traffic load on link i and $C_i$ is the capacity of link i at that state. We assume that the components change states relatively slowly so that the queueing phenomena at these links reach equilibrium and the average message delay "at a given state of the network" is meaningful.

Having defined network message delay as the reliability measure, we need an efficient way to compute it. Since there are $N^M$ network states in our model, we decide to use an approach which enumerates only the most probable states and analyzes the performance for these states. We have developed an algorithm to generate network states in order of decreasing probability for our multimode model.

## 3. Algorithm ORDER-M

There are M links and N modes for each link in the network. The modes are renamed such that the mode probabilities are in decreasing order, i.e., $p_i^0 \geq p_i^1 \geq \ldots \geq p_i^{N-1}$ for all i. Each network state S is represented by an M-vector, $S = (j_1, j_2, \ldots, j_M)$, where $j_i$ denotes the mode of link i. The most probable state is $(0, 0, \ldots, 0)$ with probability $\prod_{i=1}^{M} p_i^0$. We want to generate the m most probable states in order of decreasing probabilities. Our approach is a modification of the algorithm ORDER [9], which is designed for two-mode networks.

Let $Q = M \cdot (N-1)$. For $p_i^j$, $1 \leq i \leq M$, $1 \leq j \leq N-1$, we arrange these Q parameters in decreasing order and use $q_1, \ldots, q_Q$ to denote this ordered sequence so that each $q_k$ corresponds to a different $p_i^j$. Note that $p_1^0, \ldots, p_M^0$ are not required in the algorithm and are not included in this set of Q parameters. We first describe the way new states are generated. Let $A = \{S_a, S_b, \ldots, S_c\}$ be an ordered set of network states such that $Pr(S_a) \geq \ldots \geq Pr(S_c)$. Define an operation replace (i,j), denoted $R_i^j S_r$, as follows: $R_i^j S_r$ means that the ith element of state $S_r$ is changed to mode j and the other elements remain the same. The probability of the new state generated is easy to compute:

$$Pr(R_i^j S_r) = Pr(S_r) \cdot p_i^j / Pr(S_r(i))$$ (1)

where $S_r(i)$ is the ith element of $S_r$. This replace operation may also be defined on the whole set of states. Thus $B = R_i^j A = \{R_i^j S_a, \ldots, R_i^j S_c\}$. Note that the size of set B may be smaller than that of set A because two states $S_a$ and $S_b$ of A may differ only in element i and so are changed to the same state by $R_i^j$. In order to avoid generating duplicate new states by this replace operation, we need to record the identities of those components whose modes have already been changed in previous steps. We use a set $I_k$ to record the components used to generate new states thus far. If the ith component has been used before, then only those states in A whose ith element is 0 will be used to generate new states in $R_i^j A$. Define an operation insert, denoted $B \rightarrow A$, as follows: $C = B \rightarrow A$ = the ordered set which results when every state of B is inserted into the ordered set A. Define an operation select, denoted T(A), as follows: T(A) = the ordered set which contains the first m states of the ordered set A. The algorithm consists of two phases and Q stages with stage k corresponding to parameter $q_k$. We start with state $S_1 = (0, 0, \ldots, 0)$. To generate new states we

successively take $q_1, q_2, \ldots, q_Q$ and the corresponding $p_i^j$'s and replace the ith element of previously generated states by j. When the total number of states generated is at least m, we go to phase 2. In phase 2, we generate new states by a similar procedure, but will retain only the m most probable states.

We now give the algorithm ORDER-M:

**Phase 1:**

1. Initialize: $S_1 = (0, \ldots, 0)$, $A_0 = \{S_1\}$, $I_0 = \phi$.

2. For $k=1,2,\ldots,m-1$, repeat 3,4,5,6.

3. Find the $p_i^j$ corresponding to $q_k$.

   If $i \in I_{k-1}$, then flag=1; else flag=0

4. For $r=1, \ldots, |A_{k-1}|$, do:

   if flag=1 and $S_r(i) \neq 0$, then

   no new state is generated from $S_r$;

   else generate new state $R_i^j S_r$ and compute its probability.

   (The set of new states generated in this step is denoted $B_{k-1}$.)

5. $A_k = B_{k-1} \rightarrow A_{k-1}$ and $I_k = I_{k-1} U(i)$.

6. If $|A_k| \geq m$, go to phase 2.

**Phase 2:**

7. Initialize: $A = T(A_L)$ where L is the loop index k when we leave phase 1. The new set A is then renamed $A_L$.

8. For $k=L+1, \ldots, Q$, repeat 9,10,11,12.

9. Take $q_k$ and the corresponding $p_i^j$.

   If $i \in I_{k-1}$, then flag=1; else flag=0.

10. For $r=1, \ldots, |A_{k-1}|$, do:

    if flag=1 and $S_r(i) \neq 0$, then

    no new state is generated from $S_r$.

    else generate new state $R_i^j S_r$ and compute its probability

    (This step may be denoted $B_{k-1} = R_i^j A_{k-1}$)

11. $A = B_{k-1} \rightarrow A_{k-1}$ and $I_k = I_{k-1} U(i)$

12. $A_k = T(A)$

$A_Q$ contains, in decreasing order, the m most probable states.

The following observation on ORDER-M can be made:

Observation 1: The operations $R_i^j$, $1 \leq i \leq M$, $1 \leq j \leq N-1$, are exhaustive of the ways new states can be generated from old ones.

Lemma 1: In each stage, for any two states $S_r$ and $S_n$ which have already been generated such that $Pr[S_r] \geq Pr[S_n]$, we must have $Pr[R_i^j S_r] \geq Pr[R_i^j S_n]$ if both of these new states exist

Proof: When both $R_i^j S_r$ and $R_i^j S_n$ exist, we must have $S_r(i) = S_n(i) = 0$. It follows from Equation (1) that $Pr[R_i^j S_r] \geq Pr[R_i^j S_n]$.

Lemma 1 states that the states in $B_{k-1}$ generated in steps 4 and 10 are in order of decreasing probability. It also assures that states left out by the select operation $T(A)$ will not give rise to new states in the top m spots in the next stage.

The next theorem is a direct consequence of the above observation and lemma.

Theorem 1: At the end of ORDER-M, the m most probable states are generated.

We now estimate the time complexity of this algorithm. For each k in phase 2, $R_i^j A$ takes time $O(m \cdot M)$ and checking whether $i \in I_k$ takes time $O(M)$. In the insertion we can use a sorting algorithm with time complexity $O(m \cdot \log m)$. The select operation can be performed during the insertion, i.e., in the same sorting routine. The complexity analysis of a stage in Phase 1 is similar to Phase 2. However, instead of working with a list of m states ( the m most probable states generated thus far ), one is working with $m' < m$ states in phase 1. There are a total of Q replace operators in both phases. So the total time in the worst case is $O(Q \cdot m \cdot M + Q \cdot m \cdot \log m)$.

## 4. Coverage of the State Space

In order to estimate how many states must be generated to achieve a certain coverage of the state space, we consider the case in which $p_j^i = p_k^i$ for any mode j and links i and k. To simplify the notation, we let $p^j = p_i^j$ for all i,j and assume as before that the labeling for the states is such that $p^0 > p^1 > \ldots > p^{N-1}$. Each state of the network is represented by a random vector $(X_1, X_2, \ldots, X_M)$, where $X_i$ is the mode of the ith link, $0 \leq X_i \leq N-1$, $1 \leq i \leq M$. Note that $X_i$'s are independent random variables in our model. Define a random variable $X = \sum_{i=1}^{M} X_i$. The most probable state is $(0,0,\ldots,0)$ with $X=0$, the next most probable has $X=1$, the third most probable may have $X=1$ or 2, and so on. The most probable system states

generated by our algorithm correspond to the event $(X < A)$, where A is a positive integer. The Chernoff bound [4, 12] can be used to derive an upper bound on the probability of the set of uncovered states, i.e., $Pr(X > A)$. We first introduce some notations in the following paragraph.

The moment-generating function of $X_i$, $i=1,\ldots,M$, is $\phi(s)=E[\exp(sX_i)]=\sum_{j=0}^{N-1} p^j \exp(sj)$. The semi-invariant generating function is $\mu(s)=\ln\phi(s)$. For random variable X, the moment-generating function is $\phi_X(s)=\prod_{i=1}^{M}\phi(s)=\phi^M(s)$. Its semi-invariant generating function is $\mu_X(s)=\ln\phi_X(s)=M\ln\phi(s)$. The Chernoff bound states the following:

$$Pr(X \geq A) \leq \exp(\mu_X(s)-sA)=\exp[M\mu(s)-sA] \quad (2)$$

where the optimum value of s (which gives the tightest bound of the exponential form) is selected in accordance with

$$M\frac{d\mu(s)}{ds}=A \quad (3)$$

Differentiating the left-hand side gives

$$\frac{1}{\sum_{j=1}^{N-1} p^j e^{sj}} \sum_{j=0}^{N-1} jp^j e^{sj} = \frac{A}{M} \quad (4)$$

or

$$\sum_{j=0}^{N-1} jp^j e^{sj}= \frac{A}{M}\sum_{j=0}^{N-1} p^j e^{sj} \quad (5)$$

So selecting the optimum s involves solving a polynomial equation of degree N-1 in $e^s$.

In applying this bound to a general N-mode problem, there are some complications, however. When N > 3, the case of two components at mode 1 contributes the same value to X as the case of one component at mode 2. These two cases may have very different probabilitities. Such complications will not arise in a two-mode model, where the Chernoff bound takes the following form $Pr(X \geq A) \leq \exp[M H(A/M, 1-A/M)+(M-A)\ln p^0 + A \ln p^1]$ where $H(q,1-q)=-q\ln q-(1-q)\ln(1-q)$ and $p^0+p^1=1$. In this case, for a given coverage of the state space, say 95%, we can select A by using the above formula for $Pr(X > A) \leq 0.05$. The number of states needed is then $m=\sum_{i=0}^{A}\binom{M}{i}$. This gives a quick way of determining the value m for our algorithm. However, since the Chernoff bound is an inequality the result is pessimistic, meaning that m is usually larger than actually needed, although it is asymptotically the best among all exponential forms [12].

## 5. Delay Analysis of a Network with Multimode Links

We now apply ORDER-M to the reliability analysis of the network in Fig. 1. It has 5 nodes, M=6 links and each link has N=5 modes so Q=24. The external traffic requirements between node pairs are assumed to be Poisson with mean $\gamma_{s,t}$, where s is the source and t is the destination, $s,t=1,2,\ldots,5$. The states and the corresponding capacities of the links are given in Table 1. The associated probabilities are given in Table 2.

| | | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| | 1 | 50 | 40 | 30 | 10 | 20 |
| | 2 | 50 | 40 | 30 | 10 | 20 |
| | 3 | 50 | 40 | 30 | 10 | 20 |
| i | 4 | 50 | 30 | 40 | 20 | 10 |
| | 5 | 50 | 30 | 40 | 20 | 10 |
| | 6 | 50 | 30 | 40 | 20 | 10 |

Table 1.  $C_i^j$ in kbps  $1 \leq i \leq 6$, $0 \leq j \leq 4$

| | | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| | 1 | .99 | .006 | .002 | .001 | .001 |
| | 2 | .95 | .025 | .01 | .01 | .005 |
| | 3 | .99 | .007 | .001 | .001 | .001 |
| i | 4 | .97 | .018 | .0054 | .0036 | .003 |
| | 5 | .95 | .03 | .01 | .005 | .005 |
| | 6 | .97 | .024 | .003 | .0018 | .0012 |

Table 2.  $p_i^j$, $1 \leq i \leq 6$, $0 \leq j \leq 4$

We first arrange the 24 probabilities $p_i^j$, $1 \leq i \leq 6$, $1 \leq j \leq 4$ in decreasing order and record the (i,j) pair of the corresponding $q_k$ by a 24x2 matrix

| | | i | j |
|---|---|---|---|
| | 1 | 5 | 1 |
| | 2 | 2 | 1 |
| | 3 | 6 | 1 |
| | 4 | 4 | 1 |
| | 5 | 2 | 2 |
| | 6 | 5 | 2 |
| | 7 | 2 | 3 |
| | 8 | 3 | 1 |
| | 9 | 1 | 1 |
| | 10 | 4 | 2 |
| | 11 | 5 | 3 |
| k | 12 | 2 | 4 |
| | 13 | 5 | 4 |
| | 14 | 4 | 3 |
| | 15 | 6 | 2 |
| | 16 | 4 | 4 |
| | 17 | 1 | 2 |
| | 18 | 6 | 3 |
| | 19 | 6 | 4 |
| | 20 | 3 | 2 |
| | 21 | 1 | 3 |

| 22 | 3 | 3 |
|----|---|---|
| 23 | 1 | 4 |
| 24 | 3 | 4 |

Table 3. $q_k$, $k=1,2,...,24$.

### 5.1. Point Estimate

To analyze the mean message delay as a reliability measure of the network, algorithm ORDER-M is used to generate the m most probable states. For each state of the network, we calculate the network message delay by Kleinrock's model using a fixed routing table. Define an indicator variable $f_i$ for each state $S_i$ as follows:

$f_i = 1$ if the network delay is $\leq 200$ ms in $S_i$     (6)
    0 otherwise

then

$F = Pr$(network mean message delay is $\leq 200$ ms)

$$= \sum_i Pr(S_i) f_i$$

Here the minimum delay among all states is 106.7 ms (this value is achieved at state $S_1 = (0,0,0,0,0,0)$) and the value 200 ms is chosen as an acceptable value for the network delay. If the above summation covers all states, then we get the exact value of F. It is also possible to compute lower and upper bounds for F. Having calculated $f_i$ for the most probable states, if we let $f_i = 0$ for all other states, we get a lower bound; if we let $f_i = 1$ for all other states, we get an upper bound.

The following routing matrix is used for this example

|        |   | destination |     |     |     |     |
|--------|---|-------------|-----|-----|-----|-----|
|        |   | 1           | 2   | 3   | 4   | 5   |
|        | 1 |             |     | 1,4,3 |   |     |
|        | 2 |             |     |     |     | 2,3,5 |
| source | 3 | 3,2,1       |     |     |     | 3,5 |
|        | 4 |             |     |     |     | 4,5 |
|        | 5 |             | 5,3,2 | 5,3 | 5,4 |     |

The matrix entries represent the vertices along which the data packets are forwarded.

We generate the m=20 most probable states for this network by ORDER-M. These states and their associated probabilities are shown in Table 4. The sum of the probabilities of these 20 states is calculated to be 0.98396. The reliability measure F, the probability that the network satisfies the external traffic requirements ( i.e., the network delay does not exceed 200 ms ), is approximated as 0.92477 using these 20 states. The lower bound and upper bound for the value F as a function of m, the number of states considered, are plotted in Fig. 2

for $1 \leq m \leq 20$. We have thus illustrated the analysis of a network with multimode components and presented an indication of how reliable the network is in terms of providing a specified level of service

| rank | state vector | probability |
|------|--------------|-------------|
| 1 | (0,0,0,0,0,0) | .832264 |
| 2 | (0,0,0,0,1,0) | .026282 |
| 3 | (0,1,0,0,0,0) | .021902 |
| 4 | (0,0,0,0,0,1) | .020592 |
| 5 | (0,0,0,1,0,0) | .015444 |
| 6 | (0,2,0,0,0,0) | .008761 |
| 7 | (0,0,0,0,2,0) | .008761 |
| 8 | (0,3,0,0,0,0) | .008761 |
| 9 | (0,0,1,0,0,0) | .005885 |
| 10 | (1,0,0,0,0,0) | .005044 |
| 11 | (0,0,0,2,0,0) | .004633 |
| 12 | (0,0,0,0,3,0) | .004380 |
| 13 | (0,4,0,0,0,0) | .004380 |
| 14 | (0,0,0,0,4,0) | .004380 |
| 15 | (0,0,0,3,0,0) | .003089 |
| 16 | (0,0,0,0,0,2) | .002574 |
| 17 | (0,0,0,4,0,0) | .002574 |
| 18 | (2,0,0,0,0,0) | .001681 |
| 19 | (0,0,0,0,0,3) | .001544 |
| 20 | (0,0,0,0,0,4) | .001030 |

Table 4. The 20 most probable states
and their probabilities

### 5.2. Estimating the Distribution Function

An alternate approach to the delay analysis for our model is to find the probability distribution function of the network mean message delay. Define $F(t) = Pr$(network mean message delay is $\leq t$ ms) and an indicator variable $f_i(t)$ for state $S_i$:

$f_i(t) = 1$ if the network delay is $\leq t$ ms in $S_i$    (7)
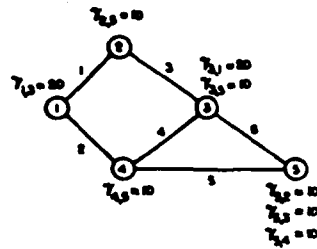    0 otherwise

then $F(t) = \sum_i Pr(S_i) f_i(t)$

The exact determination of F(t) requires the enumeration of the whole state space. However, when we generate the m most probable states, we can calculate $F_m(t) = \sum_{i=1}^{m} Pr(S_i) f_i(t)$. This value is then a lower bound on F(t), i.e., $F(t) \geq F_m(t)$ for all t. For our example, $F_m(t)$ using m=20 and the most probable states in Table 4 is calculated and plotted in Fig. 3.

### 6. Conclusions

In this paper, we have presented a multimode model for calculating network reliability and define the probability that the traffic requirements are satisfied as the reliability measure. An algorithm ORDER-M is developed which enumerates network states in order of

decreasing probability. We then apply it to compute the reliability defined above.

Using our algorithm, we are able to compute the probability distribution function of the network mean message delay to a certain accuracy in a very efficient manner. The advantage of our model is that the traffic requirement of the network is taken into account in the reliability analysis.

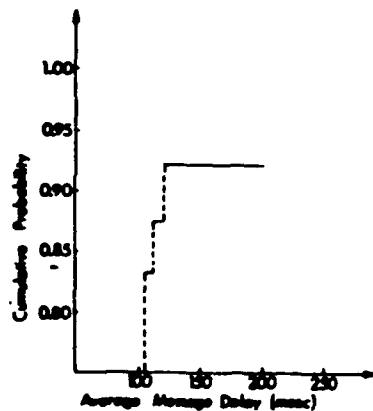Figure 1: Example Communication Network

Figure 2: Upper and Lower Bounds for F

References

1. Ball, M.O. "Computing Network Reliability." *Operations Research* 27, 4 (July-August 1979), 823-838.

2. Ball, M.O. "Complexity of Network Reliability Computations." *Networks* 10, 2 (Summer 1980), 153-165.

3. Bonaventura, V., et. al. Service Availability of Communication Networks. Proc. 1980 National Telecommunication Conference. IEEE, 1980, pp. 15.2.1-15.2.6.

4. Chernoff, H. "A Measure of Asymptotic Efficiency for Tests of a Hypothesis Based on a Sum of Observations." *Ann. Math. Stat.* 23 (1952), 493-507.

5. El-Neweihi, E., Proschan, F., and Sethuraman, J. "Multistate Coherent Systems." *J. Appl. Prob.* 15 (1978), 675-688.

6. Evans, J.R. "Maximum Flow in Probabilistic Graphs-Discrete Case." *Networks* 6 (1976), 161-183.

7. Kleinrock, L. *Queueing Systems, Vol. II.* Wiley-Interscience, 1976.

8. Kulkarni, V.G. and Adlakha, V.G. Maximum Flow in Planar Networks with Exponentially distributed Arc Capacities. Tech. Rept. UNC/ORSA/TR-84/15. University of North Carolina at Chapel Hill, December, 1984.

9. Li, V.O.K. and Silvester, J.A. "Performance Analysis of Networks with Unreliable Components." *IEEE Trans. on Commun.* COM-32, 10 (October 1984), 1105-1110.

10. Rosenthal, A. "Computing the Reliability of Complex Networks." *SIAM. J. Appl. Math.* 32, 2 (March 1977), 384-393.

11. Shacham, N., and Livne, A. An Adaptive Hybrid ARQ Algorithm for Radio Channels. Proc. ICC 85, 1985, pp. 1390-1394.

12. Wozencraft, J.M., and Jacobs, I.M.. *Principles of Communication Engineering.* John Wiley and Sons. 1965.
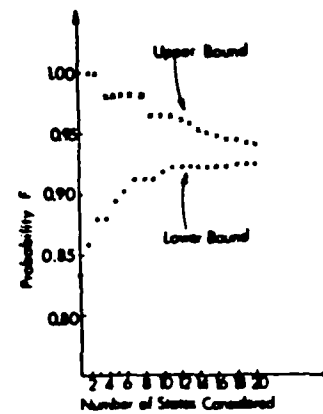
Figure 3: An Estimate of the Probability Distribution Function

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